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"It's not important what goes on each frame of film; it's the spaces between the frames that are important."

Norman McLaren
Oscar-winning animator of "Neighbors"

Physics of Timing & Spacing

This tutorial explains basic timing and spacing with simple examples, such as the acceleration of a falling ball. You'll find that the core principles apply equally well to more complex animation, such as a jumping character or a waterfall.

Ball Drop Animation Test

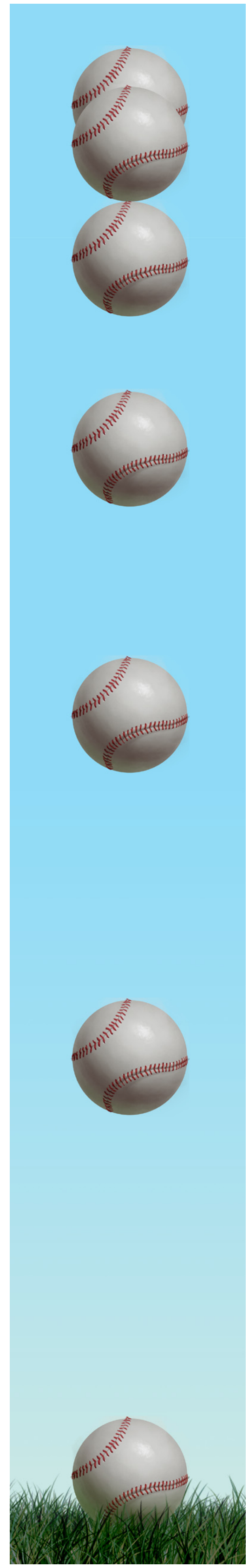
► Typically the first animation exercise you do is a falling ball. In this exercise, the drawing couldn't be simpler. It's just the same round ball in every drawing.

Nevertheless, animating the ball so that it moves realistically can still be a challenge. But the challenge is not in how you draw the ball but where it is drawn on each frame.

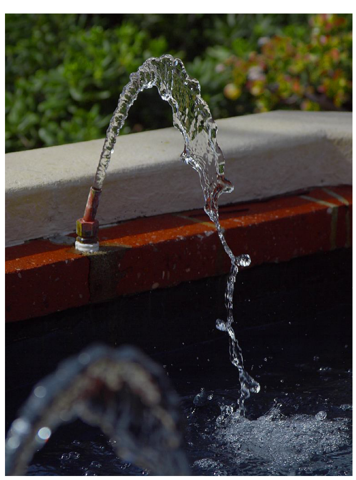
Another way to view the challenge of this test is to realize that it's all about the timing and the spacing. Suppose you space your key drawings as shown here. Are those spacings correct? Is there any pattern to the distances?

How much time does it take for the ball to move from one drawing to the next? One frame per drawing? Two? More? How does that timing depend on the size of the ball? What about the ball's weight?

In this tutorial we'll look at how to make your animation look realistic by getting the timing and spacing physically correct. You may later find that you need to alter that reality to better serve your story, but that will be easier to do if your motion looks correct from the start.



The principles illustrated in this tutorial by the ball drop animation test will apply to your other animation work, even character animation such as a jumping cat or effects animation such as a stream of water.



Frames, Keys, Clocks

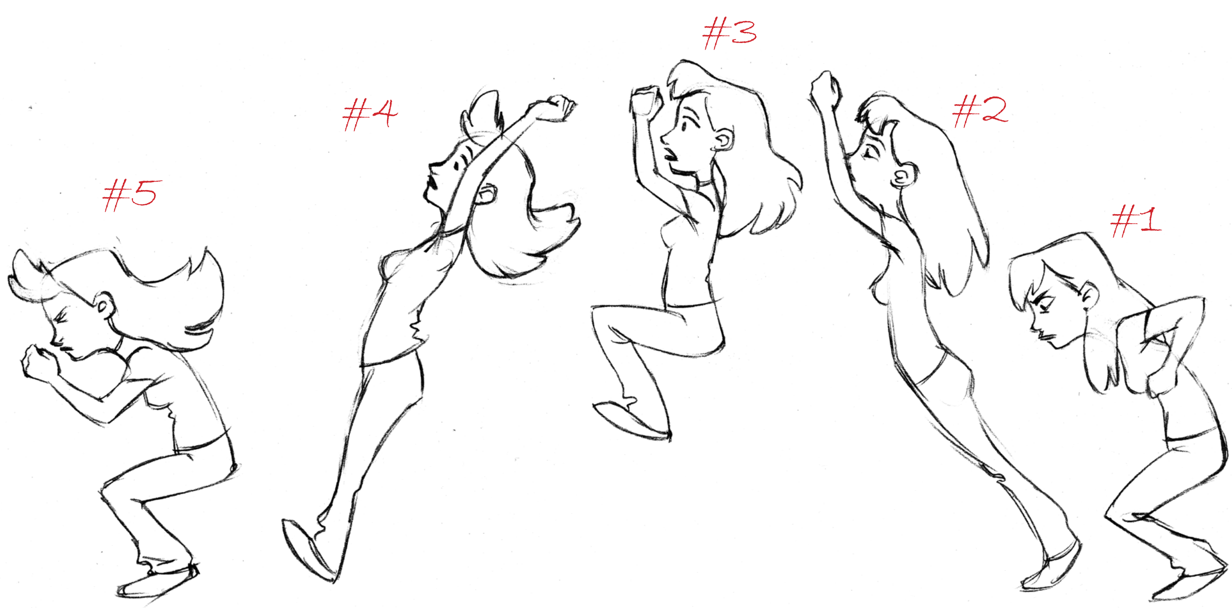
Frame	Key Pose
1	#1
2	/
3	/
4	#2
5	/
6	/
7	#3
8	/

► Animators use three different ways of measuring time:

- Frames (intervals of 1/24th of a second)
- Keys (given number of frames between key poses)
- Clocks (actual seconds as measured by a clock)

For example, you may "slug out" a scene using a stop watch, then convert that into a number of key poses, which are indicated on frames in your exposure sheet (also known as an X sheet or dope sheet).

In this example, key pose #1 is drawn on frame 1, key #2 on frame 4, and so forth. This is called "shooting on threes" since only every third frame has a different drawing. Because our persistence of vision is about a tenth of a second we see this as a (choppy) animation of a person jumping. If needed, more drawings could be added as "in-betweens."



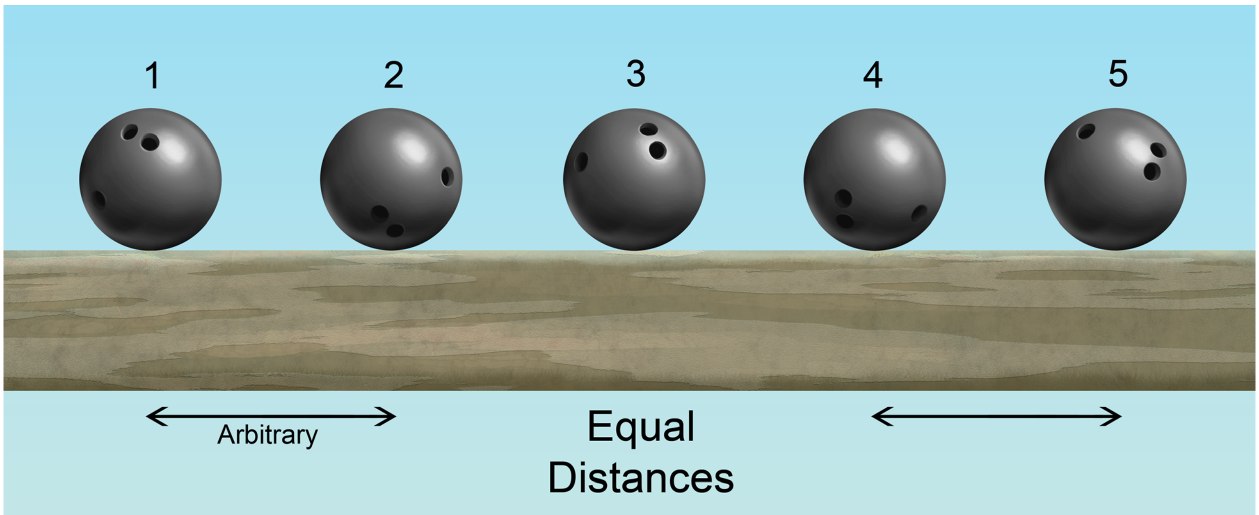
Important: In this tutorial, the number of frames between key poses is fixed in each example. In your own animation work you may find yourself putting a different number of frames between each key, such as putting more drawings when the action is fastest. But to keep things simple, in our examples we'll always have the same number of frames between each key pose and between each drawing.

Definition: 'Key poses', 'key drawings' or just 'keys' are terms used to describe those critical positions of an animated character or an object which depict the key points in its path of motion, or accents in its expression or mood.

Uniform Motion

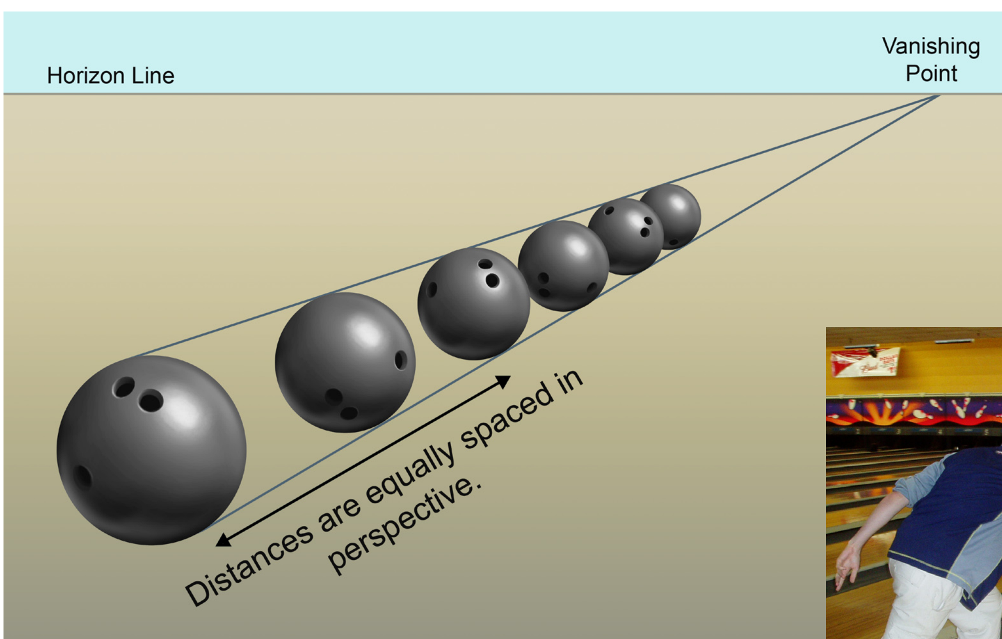
► The simplest type of motion is uniform motion; a ball rolling on a table is a good example. In uniform motion, the velocity is constant so the distance travelled from frame to frame is constant. The greater the distance, the faster the ball is rolling.

Bowling ball (one foot diameter) moving at 15 m.p.h., shot on twos.



Uniform motion may not appear uniform due to the distortion of scale when shown in perspective, such as in the example below in which a ball is rolling from the foreground towards the background.

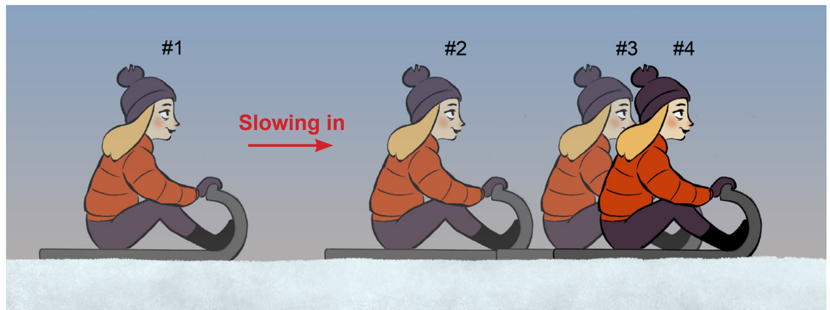
Miles per Hour	Inches per Frame
10	7 1/3
30	22
60	44
90	66



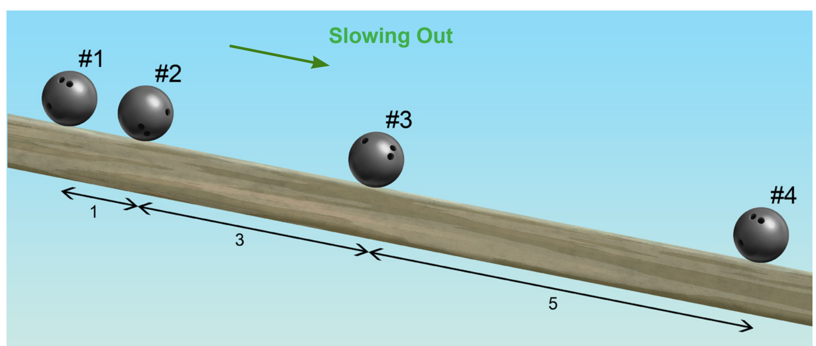
Slowing In and Slowing Out

► When an object's motion is not uniform, the object is either speeding up, slowing down, or changing direction.

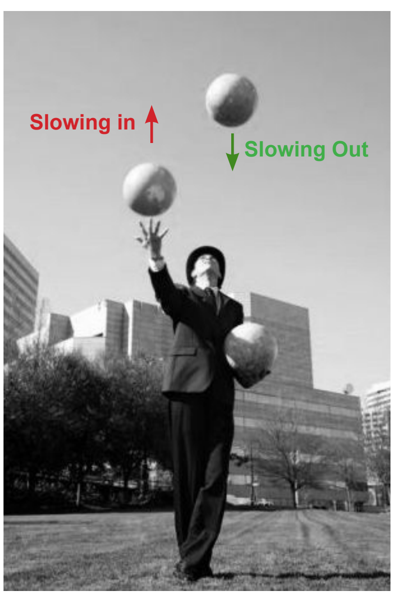
If the speed is decreasing then the spacing between drawings decreases, which in animation is called "slowing in" (or "easing in"). A sled slowing due to friction is a simple example of slowing in.



If the speed is increasing then the spacing between drawings increases, which in animation is called "slowing out" (or "easing out"). A ball rolling down an incline is a simple example of slowing out.



A ball thrown upward slows in as it rises to the apex and then slows out as it falls back downward.



Definition:

The apex is the highest point or the furthest point reached by a moving object.

Distance Fallen from the Apex

► The table below lists the distance fallen from the apex after a certain amount of time (or frames) for an object falling straight down.

Time (sec.)	Frames	Distance Fallen from Apex
1/24	1	1/3 inch
1/12	2	1 1/3 inches
1/8	3	3 inches
1/6	4	5 1/3 inches
1/4	6	1 foot
1/3	8	1 3/4 feet
1/2	12	4 feet
2/3	16	7 feet
3/4	18	9 feet
1	24	16 feet

The distances fallen from the apex may also be found using this formula:

(Distance in inches) = ($^1/_3$ inch) x (Number of Frames) x (Number of Frames)

For example, the distance fallen after 6 frames is ($^1/_3$)x(6)x(6) = 12 inches.

Question: **What is the distance fallen after 5 frames?**

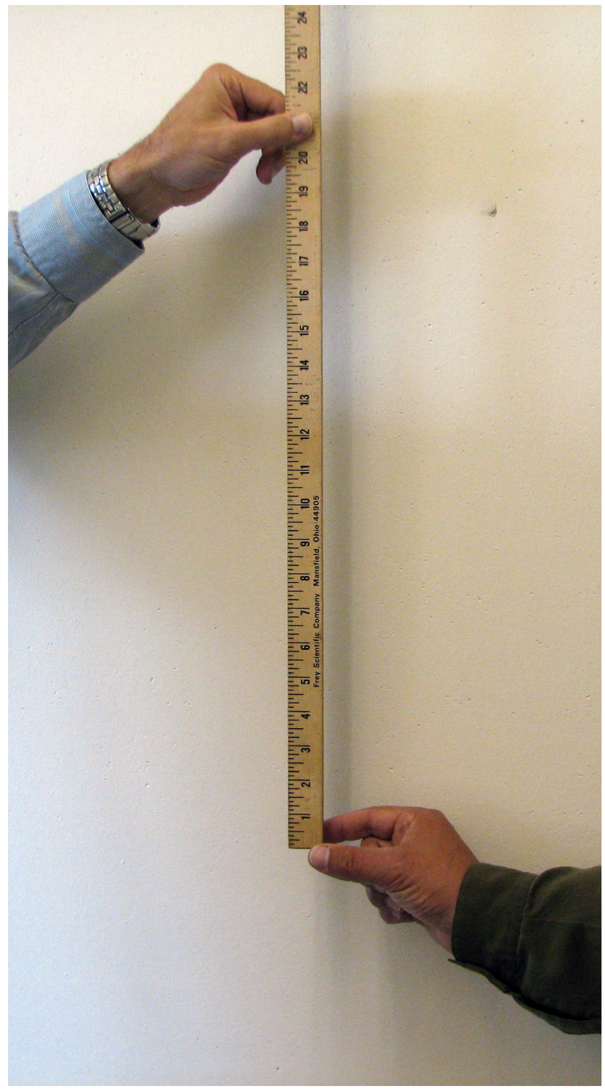
Answer: **The distance is (1/3)x(5)x(5) = 25/3 = 8 1/3 inches.**

Note: **These distances do not depend on the object's weight if air resistance is minimal. We'll discuss the effect of air resistance in another tutorial.**

Reaction Time

Distance	Time (sec.)
1"	0.07
2"	0.10
3"	0.12
4"	0.14
5"	0.16
6"	0.17
7"	0.19
8"	0.20
10"	0.23
12"	0.25
14"	0.27
16"	0.29
18"	0.30

► This table gives the time it takes an object to fall various distances. It is useful for measuring reaction time, as shown in the photo.



Experiment: Take a one dollar bill and have a friend put their thumb and index fingers near Washington's head. At random, you let go of the dollar.

Can your friend react fast enough to catch the money?

Probably not! Typical reaction time is 0.20 to 0.25 seconds.

Half the length of dollar bill is 3 inches so it takes about 1/8 of a second (0.125 seconds) to fall this distance.

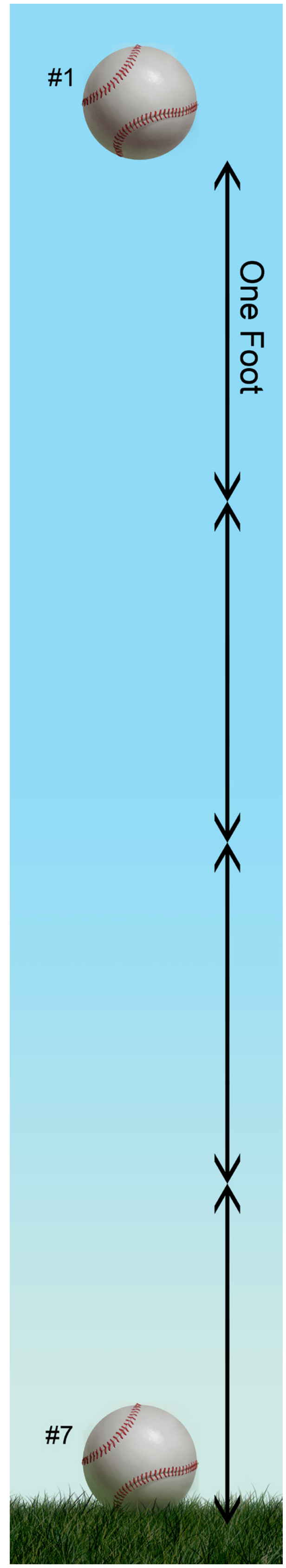
Planning a Scene (part 1)

► Suppose you want to animate a softball falling straight down from a height of four feet. Diameter of a softball is four inches.

How many total frames will you need to animate from the apex until the ball hits the ground?

The Distance Fallen table (page 6) says it takes 12 frames (½ second) for the ball to fall four feet. So there will be 13 frames, including the first frame, which is the apex.

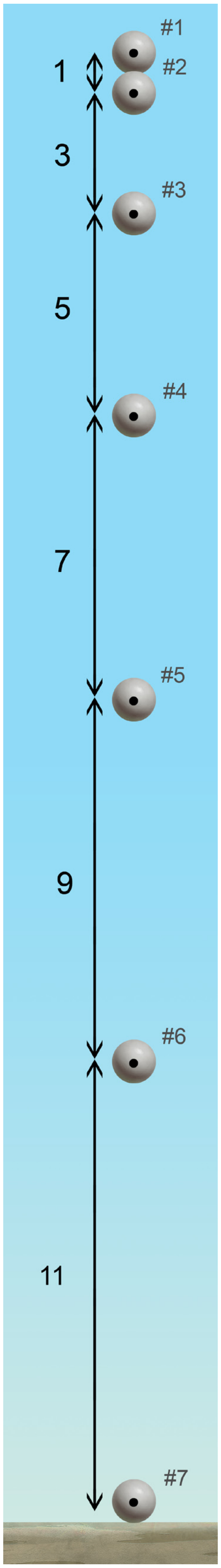
Now let's say that you'll "shoot on twos", that is, only draw every other frame. As you see from the X-sheet below, we have 7 drawings from the apex (key #1) to the bottom (#7).



Frame	Key Pose
1	#1 (apex)
2	/
3	#2
4	/
5	#3
6	/
7	#4
8	/
9	#5
10	/
11	#6
12	/
13	#7

The next question is: Where do you draw keys #2 through #6?

The Odd Rule



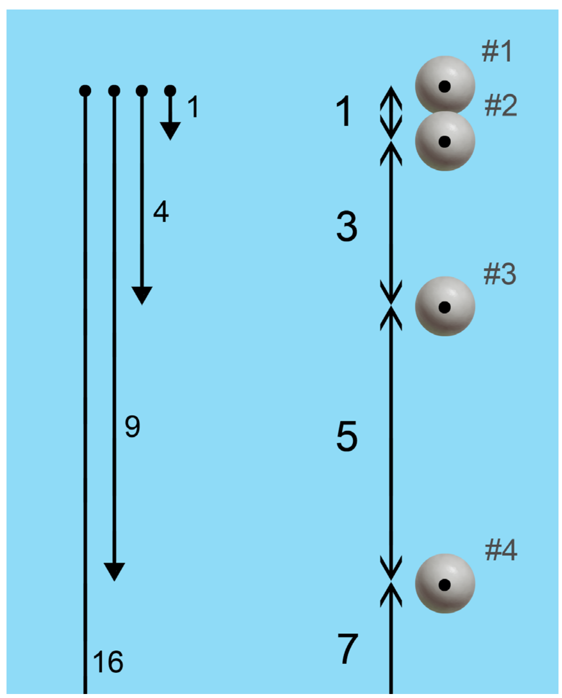
► For a falling object, the distance between drawings follows a simple pattern, which we'll call "The Odd Rule" because it uses the odd numbers (1,3,5,etc.)

From the apex, the distance between drawings increases in the ratios 1:3:5:7:9…

If you've had a physics class then you probably didn't learn about acceleration in this way.

Physicists like to measure falling by the total distance from the apex, which increases in the ratios 1:4:9:16:25:… or $1^2:2^2:3^2:4^2:5^2$:...

As you see below, the Physicists' description and the Odd Rule give exactly the same the positions for the falling ball.



Ping-pong ball falling from a height of four feet, shot on two's.

Diameter of a ping-pong ball is about 1 $^{1}/_{2}$ inches.

Planning a Scene (part 2)

► Back to our scene with a falling softball...

Remember that we're "shooting on twos" so the first drawing after the apex is when the ball has fallen for a time of two frames.

The Distance Fallen table (pg. 6) tells us that in two frames the distance fallen is $1\ ^{1}/_{3}$ inches, which happens to be a third of the diameter of the softball. Key #2 is drawn at that distance from the apex.

By the Odd Rule, the next key is spaced three times farther, which puts key #3 a distance of one diameter below key #2 (which is 4 inches below #2).

Again, by the Odd Rule, the spacing between keys #3 and #4 is five times farther than between #1 and #2. The next spacing is seven times farther, and so forth.

Question: **Instead of a softball you animate a falling bowling ball; how far below the apex is the first drawing when shooting on twos?**

Answer: **The distance is $1\ ^{1}/_{3}$ inches regardless of the object falling.**

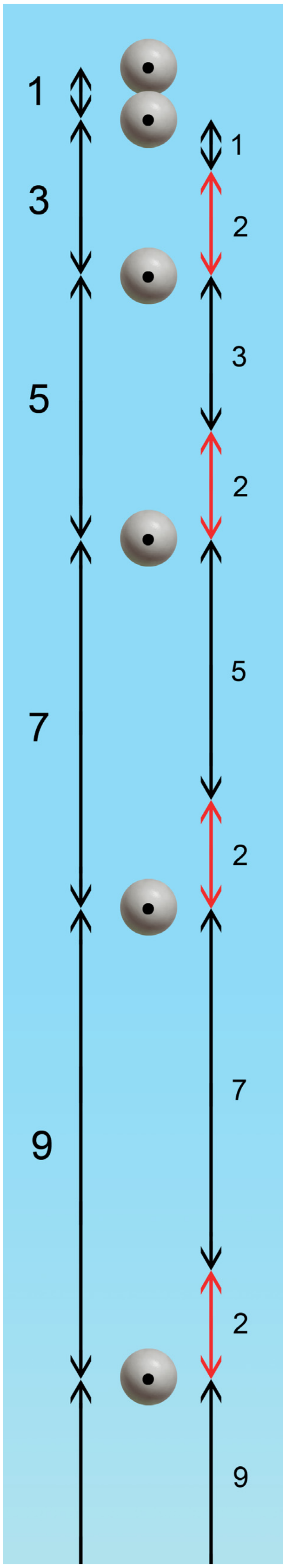
Important: **You should use the Odd Rule as a helpful guide for judging timing and spacing of your key drawings.**

Do not use a ruler and a calculator when drawing your animation since that approach only works for the very simplest tests. It is time intensive and creates stiff action.

The better approach is to understand the Odd Rule, as well as the other principles in this tutorial, and be able to apply them instinctively.

You will need such skills when creating more sophisticated animation such as a jumping cat or a stream of water.

The Odd Rule (Increment Version)



► Here's another way of thinking about the Odd Rule that may be easier to visualize and to use:

Starting from the apex, the distance between keys increases in the ratios 1:3:5:7:9…

Notice that:

$3 = 1 + 2$

$5 = 3 + 2$

$7 = 5 + 2$

$9 = 7 + 2$

etc.

In other words, after the first increment, all the others are longer by the same amount.

For any falling object, the first spacing between the drawings below the apex is:

1/3 inch	(if Shooting on Ones) or;
1 1/3 inch	(if Shooting on Twos) or;
3 inches	(if Shooting on Threes)

All the spacings between the drawings after this first one are longer by the same increment and this increment is:

2/3 inch	(if Shooting on Ones) or;
2 2/3 inch	(if Shooting on Twos) or;
6 inches	(if Shooting on Threes)

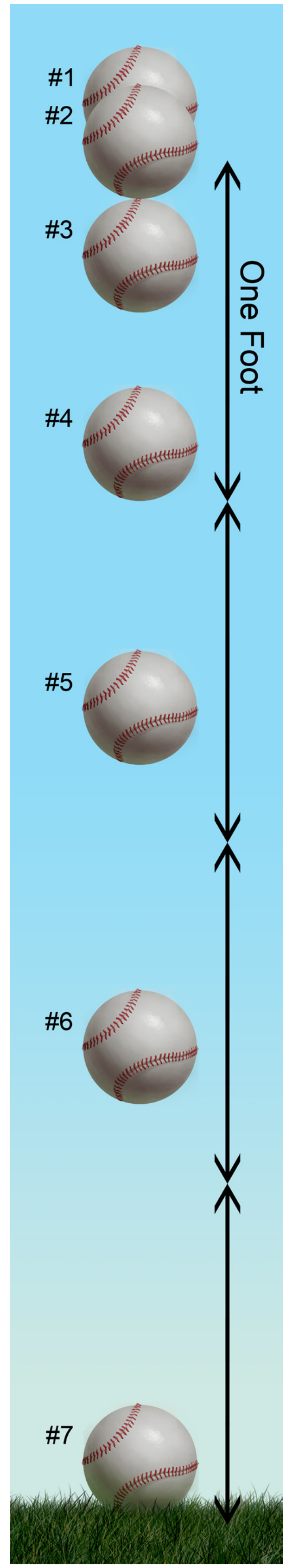
Note: **It doesn't matter whether you measure distances from the bottom, the center, or even the top of the ball as long as you always measure from the same spot on the ball.**

Planning a Scene (part 3)

► Let's finish our scene with a falling softball.

We know that the first drawing after the apex is 1 $^{1}/_{3}$ inches below it. That distance happens to be a third of the ball's diameter.

By the increment version of the Odd Rule, all the other spacings are larger by twice that amount, which is 2 $^{2}/_{3}$ inches, or two-thirds of the ball's diameter.



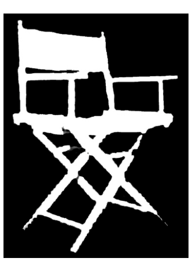
The Odd Rule essentially has three parts; the first two are the most important:

- From the apex, spacings increase between drawings.
- After the apex, that increase in the spacing is always by the same amount.

The final part of the Odd Rule is:

- That increase in the spacing is twice the distance from the apex to the first drawing below the apex.

For large objects, such as a basketball, the drawings near the apex are too close together to apply the last part. But if you increase the spacing for the rest of the drawings in a consistent fashion then the motion should still look correct.



After creating a realistic animation of a falling softball it's likely that the art director (or your teacher) will make you alter the timing or the spacing. That's life as an animator.

But your efforts were not wasted since any modifications will have the physically realistic motion as a foundation.

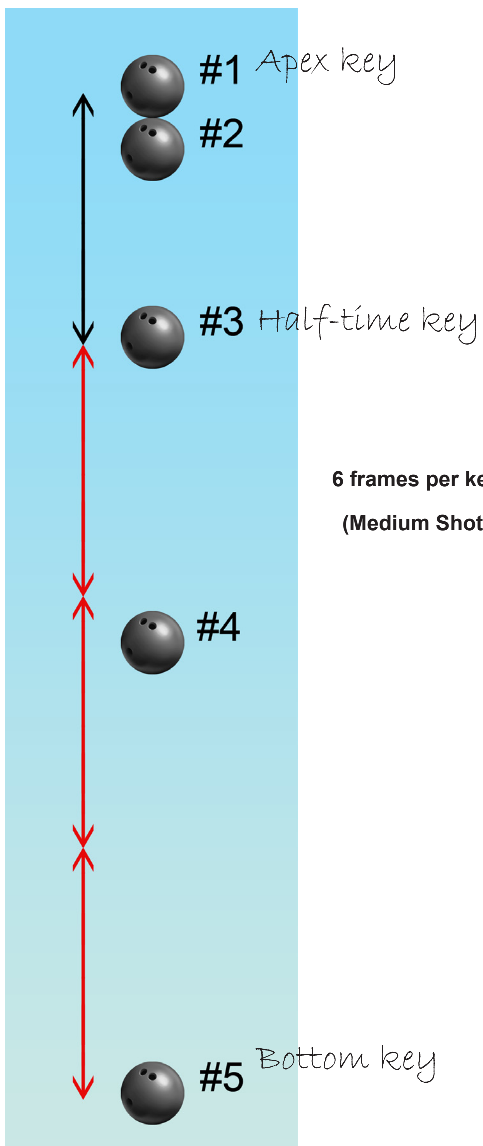
Fourth Down at Half-Time

► A useful rule for checking your spacing and timing is "Fourth Down at Half Time."

The rule is easiest to understand from the examples below:

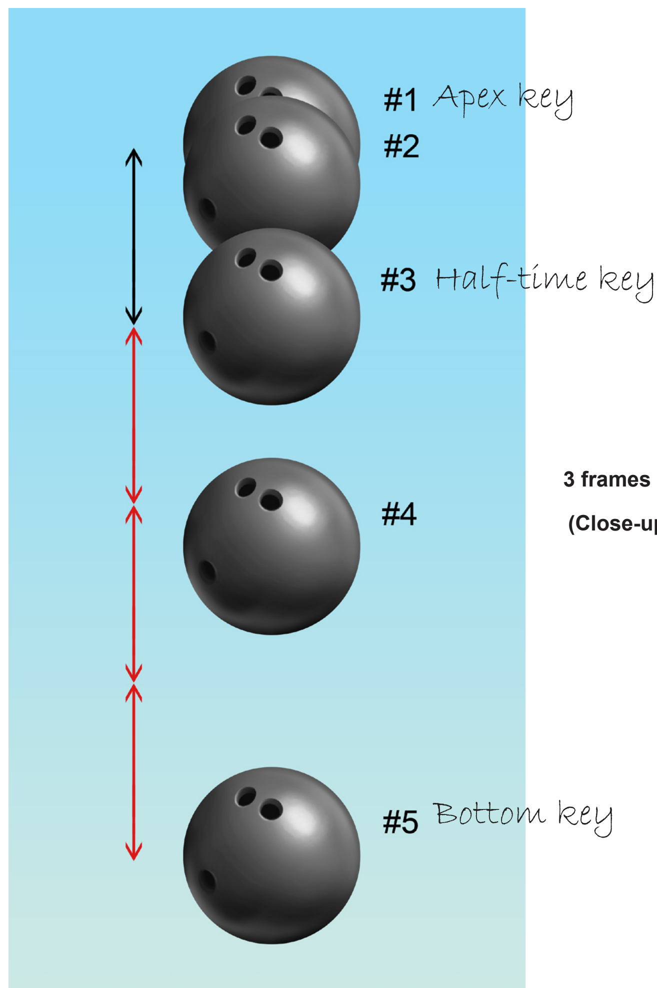
- Key #3 is half way in time between #1 and #5.
- Key #3 is a fourth the distance from #1 to #5.

The rule works like this: Pick any key below the apex, call it the "bottom key." Find the key that is half-way in time between the bottom key and the apex, call that the "half-time key." If your spacings are correct then the half-time key should be a quarter of the distance from the apex to the bottom key.



6 frames per key

(Medium Shot)



3 frames per key

(Close-up Shot)

Bowling ball is one foot in diameter

In-betweens of Accelerated Motion

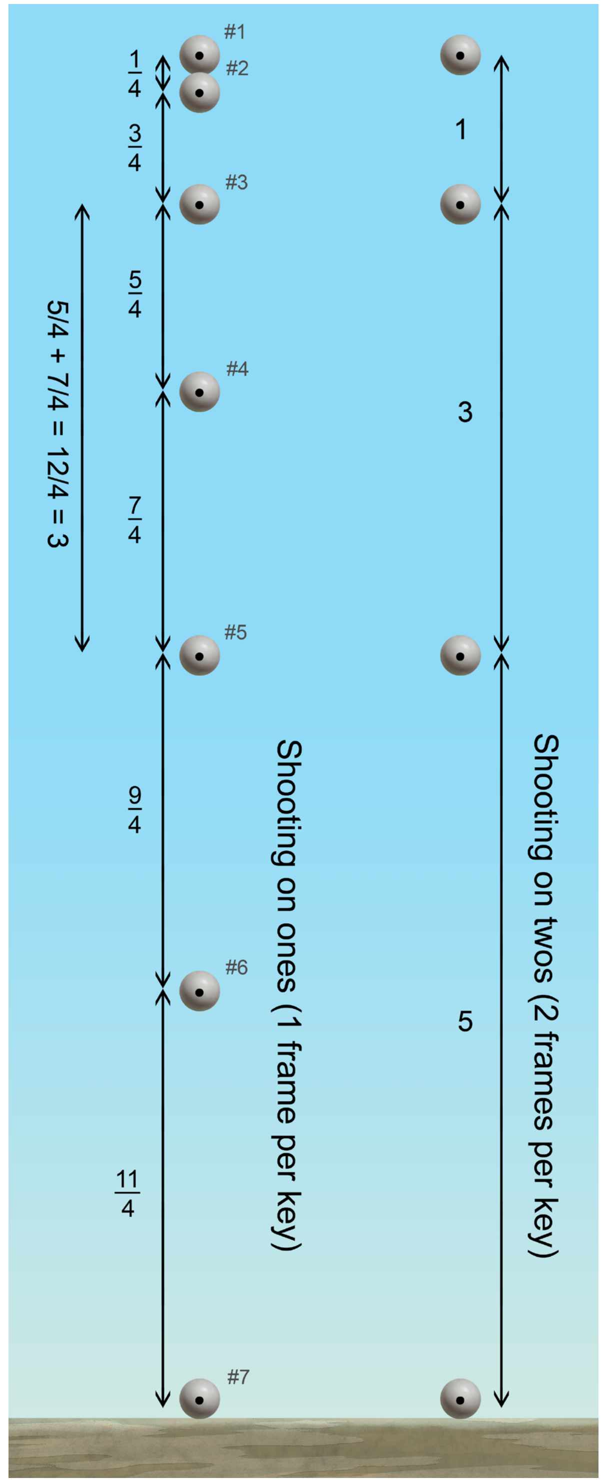
► Drawing in-betweens refers to adding extra drawings in frames in between the key pose drawings.

Shooting on twos or even on threes is adequate when the action is slow or uniform. But to capture faster action, you may need to draw on more frames.

The example on the right shows how a set of key poses shot on twos are in-betweened for shooting on ones (1 frame per key). To draw the in-betweens, divide the first distance by 4, then keep the same ratios of 1:3:5:7.

Notice that the first in-between drawing is not positioned half-way but rather it is significantly closer to the apex. On the other hand, the rest of the in-between drawings are just slightly above the half-way distance.

Finally, verify for yourself that the spacings obey the "Fourth Down at Half Time" rule.



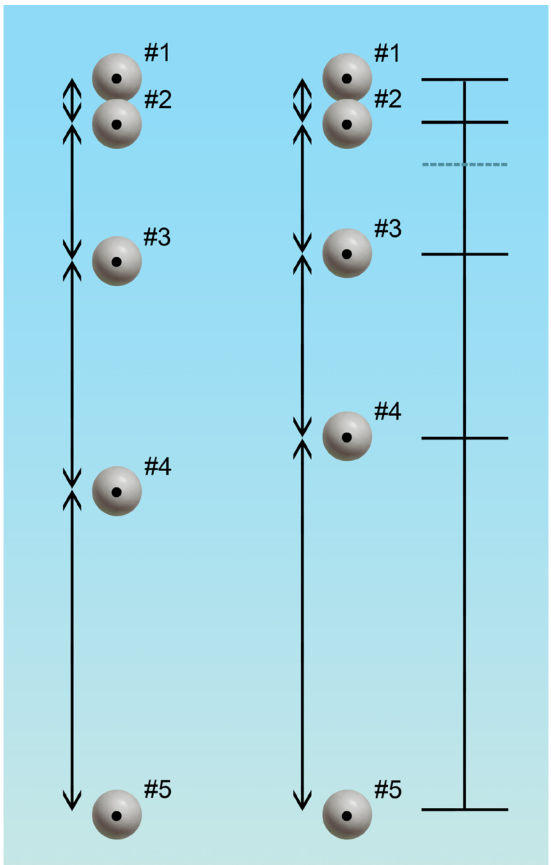
Question: **From what height is this ball falling? How big is the ball?**

Answer: **Shooting on ones it falls for 6 frames after the apex so it is falling from a height of one foot. The ball is about a third of an inch in diameter so it could be a small marble.**

Pose-to-Pose Slowing Out

► In “Straight Ahead” animation after drawing keys #1 and #2 the positions of the rest are given by the Odd Rule.

An alternative approach is “Pose to Pose” animation, in which two extreme key poses are selected and then additional drawings are added in between them.



Odd Rule Falling *a la Chai*

Dave Chai came up with an approximation for slowing out that’s easy to use in pose-to-pose animation.

Recipe for “Falling *a la Chai”*:

- Draw interval from first & last keys.
- Divide interval in half. Mark a key.
- Divide top part in half. Mark a key.
- Divide top part in half. Skip
- Divide top part in half. Mark a key.

In the example on the left, the first and last keys are #1 and #5. Following the recipe, we mark those two. Then we find the mid-point between them and mark that as key #4. Then we find the mid-point again and that’s key #3, Then find the mid-point again but skip that point. Finally, find the mid-point one last time and that’s key #2.

Although this recipe does not give exactly the correct spacing, the fact that key #4 is just a bit too high is unlikely to be noticed.

David Chai is an award-wining independent animation director, a professor of animation at San Jose State University, and one handsome fellow.

Up and Down

► We've been analyzing the timing of a ball falling straight down but what about a ball that's thrown straight upward?

It turns out that the timing of a ball slowing into the apex is just the reverse of how it slows out from the apex.

That means that from the apex you can use the same drawings twice, once on the way up (#1 to #6) and again on the way down (#8 to #13). You could do this but you shouldn't.

The reason you shouldn't re-use your drawings is that your animation will look jerky due to an optical effect called *strobing*.

When the eye sees an object disappear and then reappear in the same spot it jars the illusion that the object is moving through space, instead we perceive it as vanishing and reappearing.

To avoid strobing, shift the positions of your drawings a bit up or down; while they may not be exactly in the right locations the motion will still look realistic if you (approximately) follow the Odd Rule.

Hang time of a jump is easy to find because it's just twice the time it takes to fall from the apex height.

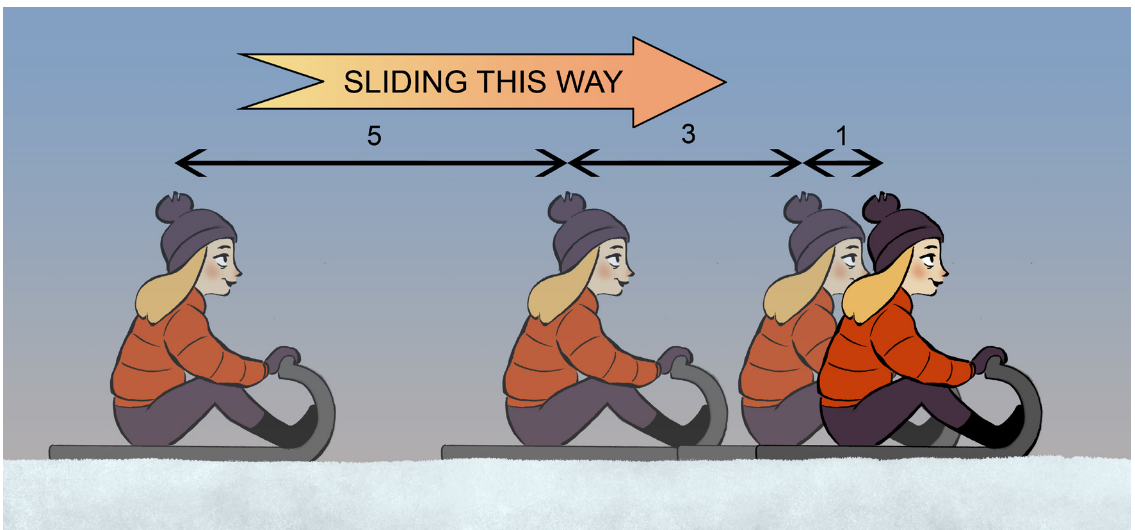
For example, the hang time for a four foot jump is 24 frames (12 frames going up and 12 coming down).

Decelerating by Friction

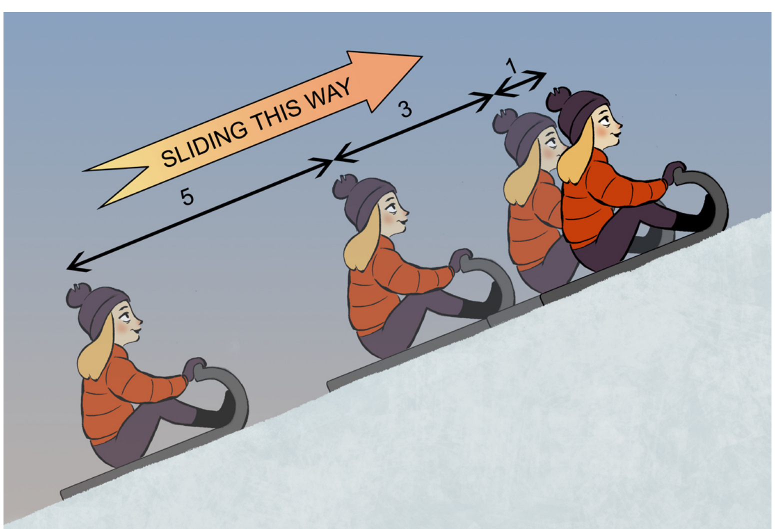
► Deceleration by friction is very similar to the deceleration of a ball thrown upward.

Using straight-ahead animation, you start by drawing the last key pose (where object stops) and go backwards in time, draw keys leading up to it with spacings in the ratios 1:3:5:7:9:…, that is, using the Odd Rule.

The increment version of the Odd Rule also applies but the distances will be different from those for a falling object.



The same procedure works for sliding uphill, but depending on the amount of friction the sliding object will either come to rest or slide back down.



Link: **The Odd Rule applies in many other situations, such as a ball rolling down a ramp or a drop of rain sliding down a window pane. In another tutorial (Creating Action) you'll see how to identify when the Odd Rule applies (and when it doesn't).**

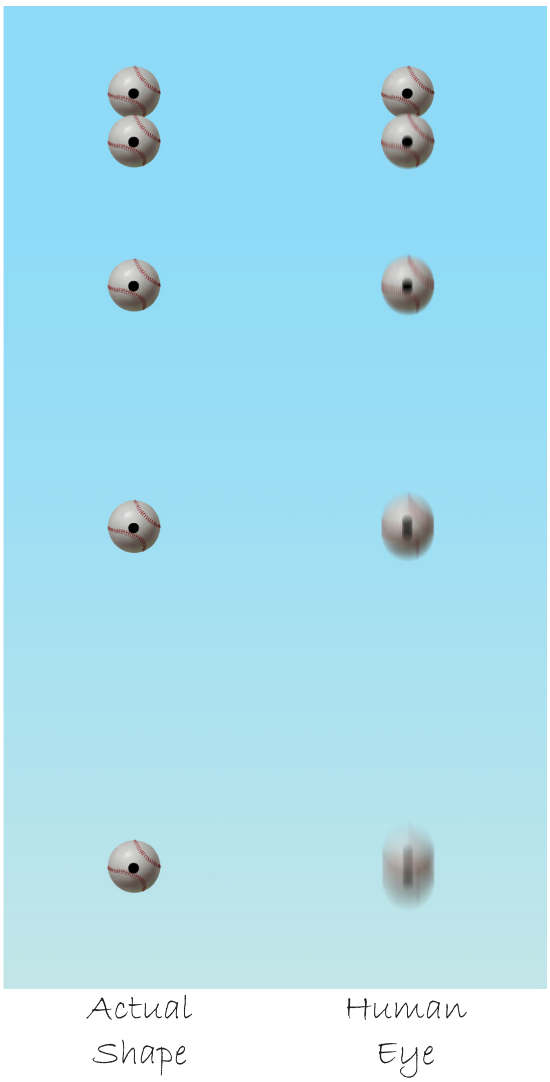
Acceleration "Stretch"

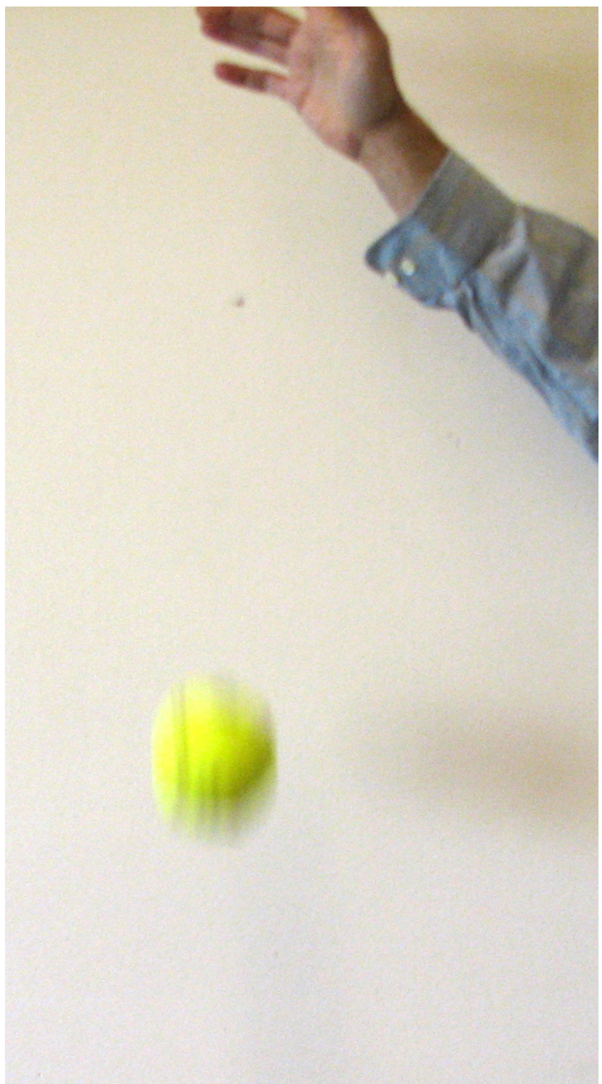
► Objects do not physically stretch as they fall (not even raindrops).

Objects visually stretch as they gain speed due to motion blur.

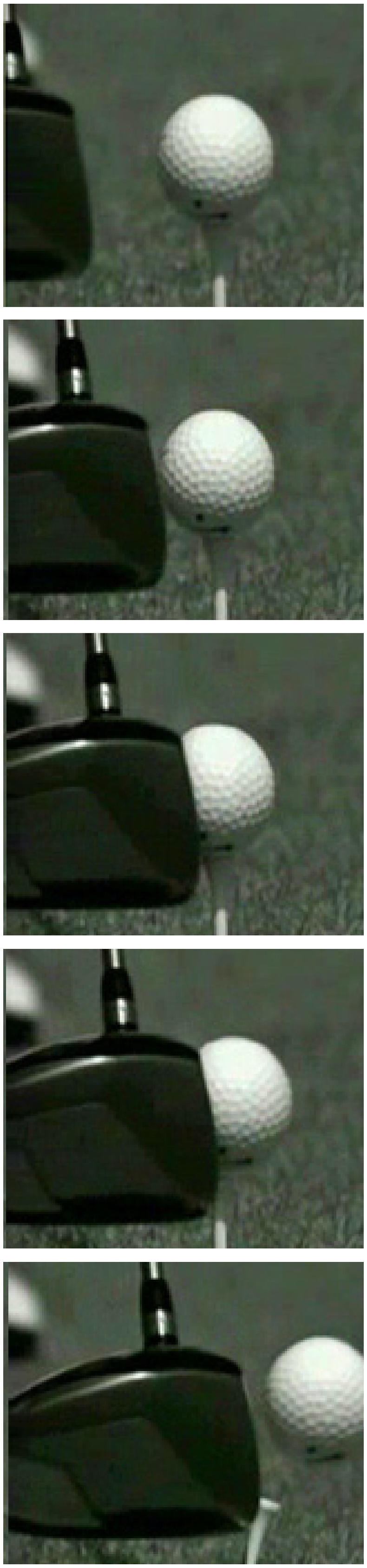
Motion blur does not depend on the object's material, however, it will look more natural for rigid objects to stretch less than elastic objects.

The motion blur makes a falling object look stretched in the vertical direction while its width is unchanged. However in animation this makes the object's volume appear to change so you should narrow the width to keep a consistent visual volume.





Bounce "Squash"



► Most objects do not physically squash except under extremely large forces of impact. For example, it takes a very fast moving golf club to produce a noticeable squash in a golf ball. You would expect this because a golf ball does not squash significantly when you squeeze it in your hands.

Although it may not be physically correct, you often want to animate objects squashing when they bounce to remove the jarring visual effect that occurs when an object suddenly changes its direction of motion.

A rigid object, such as a brick, may need little or no squash while an object such as an apple will look hard as a rock without a bit of squash.

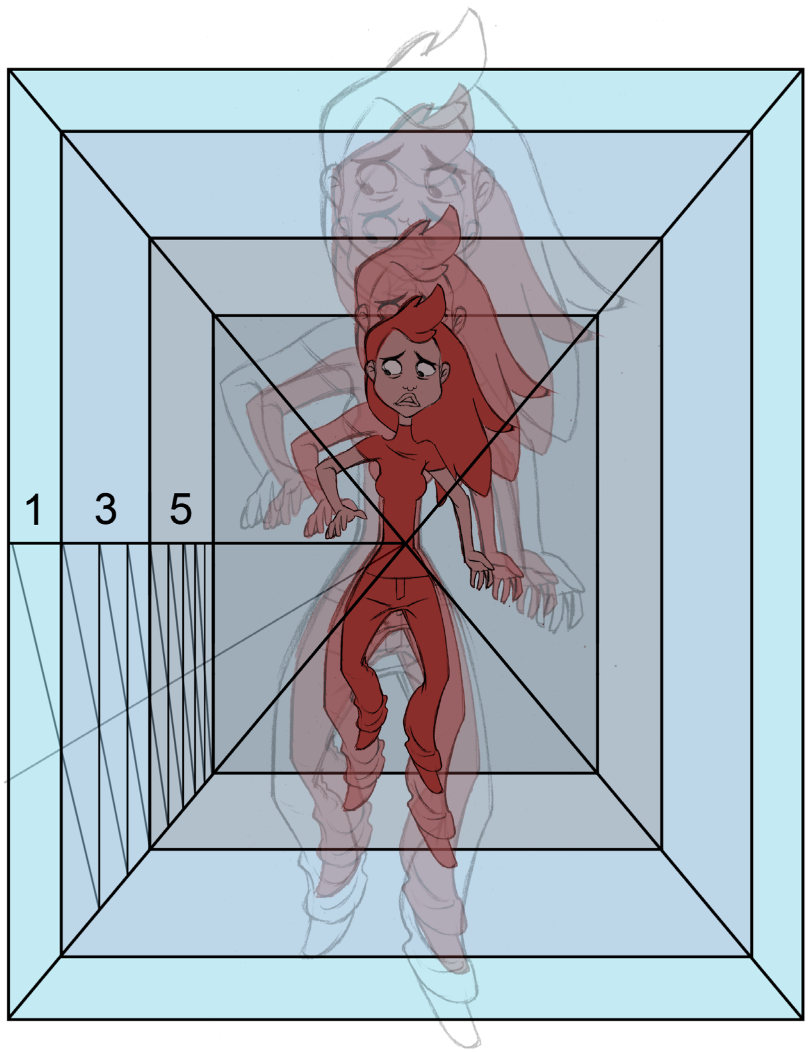
However, use squash judiciously because the more your objects deform, the less realistic (and the more cartoon-ish) they will appear.

Tip: **When drawing an object's squash, be sure to maintain consistent volume. You want your object to appear elastic; rarely do you want it to look compressible.**

Straight Falling in Perspective

► When an object falls straight down but the motion is viewed in perspective, all we have to do is use the Odd Rule but account for the distortion of sizes due to perspective.

In this example, we see Alice (in Wonderland) falling down the rabbit hole as viewed from above, looking straight down.



The size of the first two rectangles is somewhat arbitrary since they depend on the station point of the viewer (distance between the viewer and the picture plane).

Once those first key poses are selected, all the rest of the positions are fixed by the Odd Rule.

Advanced Bouncing

► This tutorial has covers the essentials of slowing in and out, focusing on the ball drop animation test. Specifically, we considered a ball falling straight up and down, focusing on the timing of a ball slowing out from (or back into) the apex.

A more advanced test would have the ball travelling in an arc, possibly in perspective, and have it lose height with each successive bounce. We'll discuss this type of motion (and more) in the next tutorial, "Physics of Paths of Action."

Image Credits

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